

Separate Answersheets are provided for Questions 4 and 5  
Please return the Answersheets with your Answerbooks

**COMP10042**

Two hours

**UNIVERSITY OF MANCHESTER  
SCHOOL OF COMPUTER SCIENCE**

Fundamentals of Computation

**Date: Tuesday 8<sup>th</sup> June 2010**

**Time: 09.45 – 11.45**

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**Please answer TWO questions from Section A worth ten marks each  
and TWO questions from Section B worth ten marks each  
and ONE question from Section C worth ten marks each**

**Use separate Answerbooks for EACH section  
and  
use the separately provided Answersheets for Questions 4 and 5 of Section B**

**Please return the Answersheets with your Answerbooks**

**This is a CLOSED book examination**

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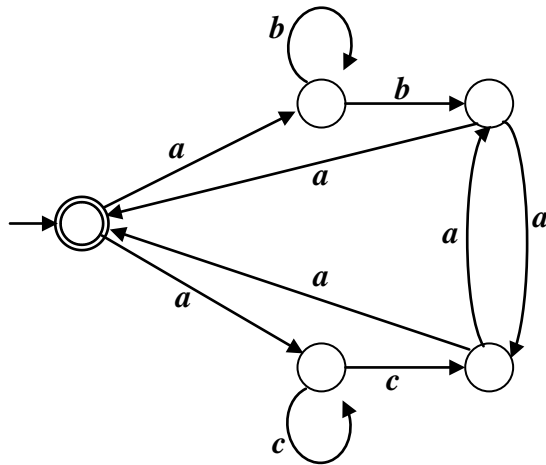
The use of electronic calculators is NOT permitted.

[PTO]

**Section A****Please answer TWO questions from the THREE available questions**

1. Consider the language of all words over the alphabet  $\{a,b,c\}$  which
- contain an even number of  $c$ s and
  - in which every  $a$  is immediately followed by  $b$ .
- a) Give a DFA for this language. (3 marks)
- b) Give a regular expression for this language. (4 marks)
- c) Give a context-free grammar for this language. (3 mark)

2. Consider the following NFA:



- a) Give a regular expression for the language accepted by this automaton. (4 marks)
- b) Turn this NFA into a DFA which accepts the same language. (6 marks)

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3. Consider the following context-free grammar:

$$S \rightarrow XaX$$
$$X \rightarrow aX \mid bX \mid \varepsilon$$

- a) Which of the following words are generated by this grammar? (2 marks)
- $a$
  - $bb$
  - $ab$
  - $ba$
- b) What is the language generated by this grammar? (2 marks)
- c) Show that this grammar is ambiguous by finding a word that has two parse trees, and give both of these. (2 marks)
- d) Give a context-free grammar for the same language that is unambiguous. (4 marks)

[PTO]

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**Section B**

**Please answer TWO questions from the THREE available questions**  
**Use the separate Answersheets provided for questions 4 and 5**

**4. Use the Answersheet for this question**

A businessman is engaged in the *organisation of cultural exhibitions*. A model of the firm is given in the answer sheet, in English, with a formalisation of some items. Formalise the remaining items. For each item:

- i) Enter the formula in the space provided underneath the English counterpart.
- ii) Use only objects (sets, relations or functions) introduced prior to the item.
- iii) Ensure the formalisation is as close to the English counterpart as possible and follow any additional instructions.

**5. Use the Answersheet for this question**

A model of a DVD rental firm is given in the answer sheet, together with some secondary features and some possible facts. Other items are added in English only; you are to formalise these. For each item:

- i) Enter the formula in the space provided underneath the English counterpart.
- ii) Use only objects (sets, relations or functions) introduced prior to the item.
- iii) Ensure the formalisation is as close to the English counterpart as possible and follow any additional instructions.

At the end, you are also to write the signature of some objects declared in the model. Do this with a *brief explanation of what the signature means*.

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6. a) The first part of the first-tier schema 'CarRally' is reproduced below. Explain the purpose of this schema, its name, the 'example variable' and how to read its content. (3 marks)

a	<u><math>r</math></u> . CarRally	All following items 'in state $r$ '
b	<i>Route description</i>	
c	$r$ .points	The geographical locations to be reached in a given sequence by the competitors
d	$r$ .start $\in r$ .points	The start of the rally, or origin
e	$r$ .end $\in r$ .points	The end of the rally, or destination
f	$r$ .suc( $i$ ) ( $i \in r$ .points \{ $r$ .end \})	The successor of $i$ , for any point $i$ different from end
g	$\forall i \in r$ .points \{ $r$ .end \} ( $r$ .suc( $i$ ) $\in r$ .points)	For every point $i$ different from end, its successor is another point
h	<u><math>r</math>.inv1:</u> $\forall i1 \in r$ .points $\forall i2 \in r$ .points ( $i1 \neq i2 \Rightarrow r$ .suc( $i1$ ) $\neq r$ .suc( $i2$ ))	Any two different points $i1$ and $i2$ must have different successors
i	<u><math>r</math>.inv2:</u> $\forall i \in r$ .points \{ $r$ .end \} ( $r$ .suc( $i$ ) $\neq r$ .start)	The point <u>start</u> is not the successor of any other point

- b) Explain the difference between a *variable feature* and a *fixed feature* or *invariant* in a schema. By way of illustration, analyse  $r$ .points,  $r$ .suc() and  $r$ .inv1 in this respect. (4 marks)
- c) Give the signature of the function  $r$ .suc, indicating in particular if it is partial or total. Briefly justify your answer by reference to the relevant items of the schema. (3 marks)

[PTO]

**Section C**

**Please answer ONE question from the TWO available questions**

7. a) Sketch the graphs of the real-valued functions  $f$  and  $g$ , where
- i)  $f(x) = 2x + 1$
  - ii)  $g(x) = \frac{1}{4}x^2 + 2$
- for the range  $x = 0$  to  $x = 4$ . (2 marks)
- b) Is there some value  $k \in \mathbb{R}$  such that for all  $x > k$ , we have  $f(x) < g(x)$ ? If so, what is the value of  $k$ ? Hint: you won't see the answer in the graphs you sketched in answer to (a) above, try solving the equation given by  $f(x) = g(x)$ . (2 marks)
- c) Give two real-valued functions, each of which grows faster than any real-valued polynomial function, such that one has a faster rate of growth than the other. (2 marks)
- d) Consider the following fragment of Java code which sorts an array  $D$ , comprising  $n$  elements, into descending order.

```
int element;
for (int i = 0; i < n-1; i++){
    for (int j = i+1; j < n; j++){
        if (D[i] < D[j]){
            element = D[j];
            D[j] = D[i];
            D[i] = element;
        }
    }
}
```

(Question 7 continues on the following page)

**Reminder:** return Answersheets for Question 4  
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(Question 7 continues from the previous page)

- i) Assume that the array  $D$  contains the six integers  
5, 3, 4, 2, 0, 1  
in the given order, i.e.  $D[0]$  is 5, etc.
- How many times is the comparison  $D[i] < D[j]$  performed?
  - How many pairs of array elements are swapped? (2 marks)
- ii) Assuming the array  $D$  contains  $n$  integer values, write down a function in terms of  $n$  that gives the maximum number of swaps that could occur. Justify your answer. (2 marks)
8. a) The “Towers of Hanoi” children’s game can be visually animated and solved by a computer program. If the program visually moves a disk from one peg to another every second, how long must the computer remain operational for a tower of 16 disks to be moved from one peg to another? If we now double the size of the tower to have 32 disks, how long will the animation now last? (4 marks)
- b) Describe a computational problem that is actually impossible to solve with a computer program. Discuss briefly whether the insolvability of some problems is a real issue for practical computer science. (6 marks)

**END OF EXAMINATION**

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**COMP10042**

**Student ID No.** \_\_\_\_\_

**UNIVERSITY OF MANCHESTER  
SCHOOL OF COMPUTER SCIENCE**

**Fundamentals of Computation**

**Question 4 and Question 5 Answersheets**

**Date: Tuesday 8<sup>th</sup> June 2010**

**Time: 09.45 – 11.45**

**Question 4**

A businessman is engaged in the *organisation of cultural exhibitions*. A model of the firm is given in the Answersheet, in English, with a formalisation of some items. Formalise the remaining items. For each item:

- i) Enter the formula in the space provided underneath the English counterpart.
- ii) Use only objects (sets, relations or functions) introduced prior to the item.
- iii) Ensure the formalisation is as close to the English counterpart as possible and follow any additional instructions.

**Background information**

The exhibition organiser will be referred to as EO. The EO has access to a vast knowledge base **kB** of artefacts (paintings, sculptures, etc.). These are **owned** by museum curators, state collection curators, private collectors etc. Each artefact in the kB has a number of attributes listed in the tables below. The creation of an exhibition on a given theme involves the following process in outline:

- a) The EO chooses a set of artefacts suitable for the exhibition's theme by performing a query on the kB and then a selection of a subset of the query's result through individual inspection.
- b) The EO seeks to obtain as many of the chosen artefacts from the owners as possible.
- c) Only some of the requested artefacts may be available. If this is insufficient, the EO carries out a second round of requests and so on until enough artefacts are obtained, if possible.

## Initial definitions

a	allArtefacts	The set of all artefacts
b	allPersons	The set of all persons, past and present
c	allPhyCats	The set of all physical categories of artefacts
d	allPhyCats = {painting, tapestry, sculpture,...}	Physical categories include painting, tapestry, sculpture,...
e	allSubjects	The set of all possible subjects of artefacts
f	allSubjects = {portrait, landscape, marine, animal, group, ship, ...}	Subjects include portrait, landscape, marine; animal, group, ship; etc.
g	allSchools	The set of all art schools (a.k.a. movements)
h	allSchools = {classic, impressionist, cubist, abstract, ...}	Schools include classic, impressionist, cubist, abstract, . . .
i	kB	The set of artefacts recorded in the knowledge base
j	$kB \subseteq \text{allArtefacts}$	Every item recorded in the knowledge base is an artefact
k	EO	The Exhibition Organiser
l	$EO \in \text{allPersons}$	The EO is a person
m	owners	The set of artefact owners
n	$\text{owners} \subseteq \text{allPersons}$	Artefact owners are persons

## Further features

a	The physical category of an artefact $a$
	phyCat( $a$ ) ( $a \in \text{allArtefacts}$ )
b	The physical category of an artefact $a$ belongs to the set of all physical categories
	$\forall a \in \text{allArtefacts}$ (phyCat ( $a$ ) $\in$ allPhyCats)
c	The creator of an artefact $a$
	creator( $a$ ) ( $a \in \text{allArtefacts}$ )

d	The creator of an artefact $a$ belongs to allPersons

(½ mark)

e	The set of subjects of an artefact $a$
	subjects( $a$ ) ( $a \in \text{allArtefacts}$ )

f	The set of subjects of an artefact $a$ is a <i>subset</i> of the set of all subjects

(½ mark)

g	The school of the artefact
	school( $a$ ) ( $a \in \text{allArtefacts}$ )

h	The school of an artefact $a$ belongs to the set of all schools

(½ mark)

i	The owner of an artefact $a$
	owner( $a$ ) ( $a \in \text{allArtefacts}$ )

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

j	The owner of an artefact $a$ belongs to the set of all artefact owners

(½ mark)

k	The set arts1 of artefacts selected by a query on the knowledge base
	$\text{arts1} \subseteq \text{kB}$

l	<i>Possible query:</i> arts1 is the set of all paintings created by Renoir and whose subjects include a group and a landscape.

(1 mark)

m	Every artefact $a$ belonging to arts1 is from the impressionist school

(1 mark)

n	The set arts2 of artefacts individually selected from arts1, thus forming the total set of requested artefacts
	$\text{arts2} \subseteq \text{arts1}$
o	The set of artefacts requested from an owner $p$
	$\text{reqFrom}(p)$ ( $p \in \text{owners}$ )

p	For any owner $p$ , the artefacts requested from $p$ are those belonging to arts2 and owned by $p$

(1 mark)

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

q	The set of relevant owners
	rlvOwners

r	The set of relevant owners is the set of owners of whom one or more artefacts are requested

(1 mark)

s	The set of artefacts 'agreed by an owner $p$ ', i.e. requested from $p$ and whose request has been accepted by $p$
	agreedBy( $p$ ) ( $p \in$ owners)

t	For any owner $p$ , the set of artefacts agreed by $p$ is a subset of the artefacts requested from $p$

(1 mark)

u	The set of artefacts obtained from all owners
	arts3

v	<i>Use generalised union</i> The set of artefacts obtained from all owners is the union of the sets of artefacts agreed by their respective owners

(1 mark)

w	No requested artefact belongs to more than one owner

(1 mark)

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

x	<p>If only 3 owners have agreed to lend 1 or more Renoir paintings and <u>arts3</u> contains 4 such paintings then at least one owner must be willing to lend more than one Renoir painting</p> <p><i>Use the function <u>card</u> and the set</i></p> <p><math>RP = \{a \in \text{allArtefacts} \mid \text{phyCat}(a) = \text{painting} \wedge \text{creator}(a) = \text{Renoir}\}</math></p>

(1 mark)

(Total (max 10): \_\_\_\_\_)

## Question 5

A model of a DVD rental firm is given below, together with some secondary features and some possible facts. Other items are added in English only; you are to formalise these. For each item:

- i) Enter the formula in the space provided underneath the English counterpart.
- ii) Use only objects (sets, relations or functions) introduced prior to the item.
- iii) Ensure the formalisation is as close to the English counterpart as possible and follow any additional instructions.

At the end, you are also to write the signature of some objects declared in the model. Do this with a *brief explanation of what the signature means*.

a	The clients of the DVD rental firm
	clients
b	The dvds owned by the firm
	dvds
c	The dvd $d$ is let to client $c$ (where $d$ is any dvd and $c$ any client of the firm)
	$d$ letTo $c$ ( $d \in$ dvds, $c \in$ clients)
d	Hirer of dvd $d$
	hirer( $d$ ) ( $d \in$ dvds)

e	For any dvd $d$ and any client $c$ , $c$ is the hirer of $d$ iff $d$ is let to $c$

(½ mark)

f	Dvds hired by client $c$
	dvdsHiredBy( $c$ ) ( $c \in$ clients)

g	For any client $c$ , the dvds hired by $c$ are the dvds let to $c$

(½ mark)

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

h	Dvds hired
	dvdsHired

i	The dvds hired are the dvds each of which is let to some client

(½ mark)

j	The set of all possible topics
	allTopics
k	Topics of dvd $d$
	topics( $d$ ) ( $d \in \text{allDvds}$ )

l	The topics of any dvd $d$ is a <i>subset</i> of all possible topics

(½ mark)

m	The set of all languages of the world
	allLanguages
n	The languages of a dvd $d$ <span style="float: right;"><i>Note: a dvd may be in several versions</i></span>
	langs( $d$ ) ( $d \in \text{allDvds}$ )

o	The languages of a dvd are languages of the world

(½ mark)

p	The (main) language spoken by any client $c$
	lang( $c$ ) ( $c \in \text{clients}$ )

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

q	The (main) language of any client $c$ is a language of the world

(½ mark)

r	For any client $c$ and any dvd $d$ hired by $c$ , $c$ speaks one of the languages of $d$

(1 mark)

s	Any dvd $d$ let to any client $c1$ is not let to a <i>different</i> client $c2$

(1 mark)

t	Every dvd with a Spanish version and war as a topic has been hired

(1 mark)

u	If Deby has hired dvd $d01$ then there is a client $c$ who has hired a dvd $d$ and who does not speak any of the languages of $d$

(1 mark)

(Running total carried forward: \_\_\_\_\_)

(Running total carried forward: \_\_\_\_\_)

v	For every dvd of the firm on the topic of animals, the firm has more than ten dvds each of which covers both the topics of gambling and stress

(1 mark)

w	Every dvd covering the topics of honour <i>and</i> betrayal and with a French <i>or</i> a German version has been hired

(1 mark)

x	Signature of topics with brief explanation

(1 mark)

(Total (max 10): \_\_\_\_\_)