Two hours

A separate Answersheet is provided for Question 4
Please return the Answersheet with your Answerbooks

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Fundamentals of Computation

Date: Monday 23rd May 2011

Time: 09:45 - 11:45

Please answer any TWO questions from Section A
Please answer Question 4 and ONE other question from Section B
Please answer any ONE question from Section C

Use separate Answerbooks for EACH section
and
use the separately provided Answersheet for Question 4 of Section B

Please return the Answersheet with your Answerbooks

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

Section A

Answer *two* of the three questions

1. a) Do the following regular expressions define the same language? Justify your answer.

(2 marks)

$$(01)^*0 \qquad \text{and} \qquad 0(10)^*$$

b) Give a DFA over the alphabet $\{a,b,c\}$ which accepts precisely the words which

- contain at least one a and
- whose number of cs is divisible by 3. (4 marks)

c) Give a regular expression for the language of all those words over the alphabet $\{0,1\}$ which contain at most one copy of the string 00.

Note that the string 000 is not in that language since it can be thought of as $(00)0$ and $0(00)$ which shows that it contains two copies of that string. (4 marks)

2. Give a DFA for the language of all words over the alphabet $\{a,b,c\}$ that match the pattern $(a|b)^*a|(ab)^*b$. (10 marks)

3. a) Give a context-free grammar for the language of all words over the alphabet $\{a, b, c\}$ for which the number of letters *a* is equal to the number of letters *b* and *c* combined.
(3 marks)

b) Give a parse tree following your grammar from a) for the string *cbabaa*.
(2 marks)

c) Is it possible to give a DFA for the language from a)? Support your answer with a reason.
(2 marks)

d) Is it possible to give a regular expression for the language of all words over the alphabet $\{0, 1\}$ which

- contain the same number of 0s and 1s and
- in which no two adjacent symbols are the same?

Support your answer with a reason.
(3 marks)

Section B

Answer *Question 4* and *one* of the remaining two questions

Use the separate Answersheet provided for Question 4

4. **Use the Answersheet for this question**

Remember to return your Answersheet for Question 4

5. a) The schema 'BookClub' is reproduced from the Part 2 lecture notes as follows:

3.2.1A	
<u>c: BookClub</u> c.books $\subseteq$ allBooks c.mates $\subseteq$ allPersons b c.onLoanTo m $(b \in c$.books, $m \in c$.mates$)$ <u>c.inv1 :</u> $\forall b \in c$.books $\forall m1 \in c$.mates $\forall m2 \in c$.mates $[(b\ c.\text{onLoanTo}\ m1 \wedge m1 \neq m2)$ $\Rightarrow \neg(b\ c.\text{onLoanTo}\ m2)]$	 The books of c The mates of c Book b is on loan to mate m in c <u>Invariant 1:</u> For any book b of c if b is on loan to a mate $m1$ of c then b is not on loan to a different mate $m2$ of c

Consider the following declaration of three states $c1$, $c2$, $c3$: BookClub of a particular book club.

i. Define the notion of a *variable feature* of any state c:BookClub.

ii. List the *variable features* of state $c1$.

iii. Assume books are referred to as $B1$, $B2$, etc., i.e. allBooks $= \{B1, B2 \ldots\}$. Illustrate your answer to (i) by constructing suitable instances of $c1$.books and $c2$.books and relating these two features appropriately.

(4 marks)

b) Establish an operation InsM on BookClub, by filling the gaps marked by '. . . ' in the following operation schema.

<u>InsM</u>
Insert a new person into the set of mates of a book club. Given c : BookClub and p : allPersons where (pre 1) . . . InsM$(c,p) = c'$ where c' : BookClub and (1) . . . (2) . . . (3) . . .

(3 marks)

(Question 6 continues on the following page)

Remember to return your Answersheet for Question 4

c) Establish an operation RemM on BookClub, by filling the gaps marked by '...' in the following operation schema. Take care to specify the precondition (pre 1) properly so that the operation is *as simple as possible*. (Hint: in formulating your precondition, you may use the secondary feature c.booksBorrowedBy(m) which is the set of books borrowed by any m : c.mates in any state c : BookClub.)

<u>RemM</u>
Remove a mate from the set of mates of a book club.
 	
Given c' : BookClub and m : c'.mates where
    (pre 1)   ...
RemM$(c', m) = c$ where c : BookClub and
    (1) ...
    (2) ...
    (3) ...

(3 marks)

Remember to return your Answersheet for Question 4

6. a) A portion of the first-tier schema ‘CarRally’ is reproduced below, slightly modified and with new feature indexes. Briefly describe features (i), (iii) and (iv), indicating for each whether it is: a *set*, or a *function*, or a *relation*, or a *condition*; and whether it is a *variable feature* or a *fixed feature*. For any function or relation, give its *signature*.

	r : CarRally (simplified)	**All following items ‘in state r’**
	Competitors and teams	
(i)	r.competitors	The rally’s competitors
(ii)	r.teams	The rally’s teams
(iii)	r.mates(t) $(t \in r$.teams$)$	The mates of team t
(iv)	r.inv1 : $\forall t \in r$.mates r.mates$(t) \subseteq r$.competitors	For any team t, all the mates of t are competitors
(v)	r.inv2 : $\forall t1 \in r$.teams $\forall t2 \in r$.teams $(t1 \neq t2$ $\Rightarrow$ r.mates$(t1) \cap r$.mates$(t2) = \emptyset)$	Any two different teams $t1$ and $t2$ have no mates in common

(4 marks)

b) Establish an operation Emp for the above schema. The purpose of Emp is to return the ‘smallest’ r : CarRally, in a sense you must briefly explain. Your specification must satisfy the *minimality principle* for such constructions. Briefly justify your answer in this respect. (3 marks)

c) Considering only the features listed in the above schema, establish an operation InsM, which given r : CarRally, a team t of r and a competitor c of r, inserts c into the set of mates of the team t. Include all necessary preconditions if any.

(3 marks)

Remember to return your Answersheet for Question 4

Section C

Answer *one* of the two questions

7. a) On the same plot, carefully sketch the graphs of the real-valued functions f and g, where

   i. $f(x) = 3x + 1$
   ii. $g(x) = \frac{1}{2}x^2 + 4$,

   for the range $x = 0$ to $x = 4$.

   (2 marks)

   b) Is there some value $k \in \mathbb{R}$ such that for all $x > k$, we have $f(x) < g(x)$? If so, what is the value of k. Hint: you won't see the answer in the graphs you sketched in answer to (a) above, so solve the equation given by $f(x) = g(x)$. (2 marks)

   c) Give an example of a real-valued function that grows faster than any real-valued polynomial function. (1 mark)

   d) Consider the following fragment of Java code which uses a binary search to find a given value `v` in an ordered array `a`, comprising `n` elements.

```
int lb = -1;
int ub = n;
int probe;
boolean found = false;
do {
  probe = (ub - lb)/2;
  if (probe == 0) break;
  probe = probe + lb;
  if ( a[probe] == v ) found = true;
  else
    if ( a[probe] < v )
      // search the right part
       lb = probe;
    else
      // a[probe] must be greater then v
      // search the left part
      ub = probe;
}
while ( !found );
```

(Question 7 continues on the following page)

For parts i) and ii) below, assume the array `a` contains just the following `15` elements, in the given ascending order

```
1, 4, 5, 7, 11, 17, 20, 22, 29, 30, 31, 37, 44, 47, 48
```

i) What and how many comparisons (of an array element against the desired value) are performed to find the value `17`? (1 mark)

ii) How many comparisons are performed to fail to find the value `33`? Justify your result. (1 mark)

iii) Suppose the array `a` now contains 1024 elements, what is the maximum number of comparisons that might be performed to find some given given element? (1 mark)

iv) Now consider the general case when the array has `n` elements in ascending order. Write down, and informally justify, a mathematical expression in terms of `n` that gives the maximum number of 'probes' that might be necessary to find a given value? (2 marks)

8. a) Describe the Travelling Salesman Problem (TSP) and explain, informally, why it is classed as a very hard, indeed, intractable, problem. (3 marks)

b) Explain what is meant by the term *problem size*. What is used as the measure of problem size for the TSP? (2 marks)

c) Explain a "brute force" method for obtaining a solution to the problem, then give and informally justify its worst case time performance measure in terms of the problem size (given in answer to part 8b above)? (2 marks)

d) A computer, rated at 100 watts, can process 10^8 tours a second. Assuming the cost of electricity is 10p per kilowatt hour (an hour at 1000 watts), give an estimate for the most it could cost to solve, by your brute force method, a TSP with problem size 21. (You may assume that 20! is approximately 2.43×10^{18}.) (3 marks)

END OF EXAMINATION

COMP11212 **Student ID No. ________________________**

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Question 4 Answersheet

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After completing the present Answersheet, please attach it to your Answerbook.

Question 4

Introduction

HandyCo is a business company offering a maintenance service for office blocks. The service covers plumbing, decorating, carpentry, etc.

When a job arises, the worker assigned to it borrows the necessary tools from the company's tool shop for the duration of the job. The tools are individually identified.

A model of the firm is given below in English, with a formalisation of some items. Formalise the remaining items. For each item:

i) Enter the formula in the space provided *underneath* the English counterpart.

ii) Use only objects (sets, relations or functions) introduced *prior* to the item.

iii) Ensure each formula is as close to its English counterpart and as consistent with previous formulae as possible. Follow any additional instructions. These are typed in italics, close to the statements which they concern.

Initial definitions: predefined sets

	The set of all possible tools
	allTools
	The set of all possible persons
	allPersons
	The set of all physical categories of tools
	allPhyCats
	Physical categories include hammer, saw, spanner, (paint) brush, . . .
	allPhyCats = {hammer, saw, spanner, brush, . . . }
	The set of all possible trades involved in building maintenance: plumbing, decorating, carpentry, . . .
	allTrades = { plumbing, decorating, carpentry, . . . }

Primary features of the firm

	The set of tools of the firm
	tools
	The set of workers of the firm
	workers
	Tool *t* is lent to worker *w*
	t lentTo *w* ($t \in$ tools, $w \in$ workers)
	No tool may be lent to more than one worker
	$\forall t \in$ tools $\forall w1 \in$ workers $\forall w2 \in$ workers [(t lentTo $w1 \wedge w1 \neq w2) \Rightarrow \neg (t$ lentTo $w2$)]
	Trade of a worker
	trade(*w*) ($w \in$ workers)
	$\forall w \in$ workers (trade(w) $\in$ allTrades)
	Physical category of a tool
	phyCat(*t*) ($t \in$ tools)
	$\forall t \in$ tools (phyCat(t) $\in$ allPhyCats)

Possible facts based on primary features

1	**The firm has at least one worker whose trade is plumbing**

(1 mark)

2	**Every tool has been borrowed by some worker**

(1 mark)

3	**There is a worker who has borrowed every tool**

(1 mark)

(Running total carried forward: _________)

(Running total carried forward: ________)

Some secondary features

	Borrower of tool *t*
	borrower(*t*) (*t* ∈ tools)

4	**For any tool *t* and any worker *w*, *w* is the borrower of *t* iff *t* is lent to *w***

(1 mark)

	Tools borrowed by worker *w*
	toolsBorrowedBy(*w*) (*w* ∈ workers) *You may use the abbreviation: toolsBorBy(w)*

5	**For any worker *w*, the tools borrowed by *w* are the set of tools lent to *w***

(1 mark)

6	**Every worker whose trade is painting is not a plumber**

(1 mark)

7	**If Bill has borrowed every tool then any worker other than Bill has borrowed no tools** *Use toolsBorrowedBy (or its abbreviation toolsBorBy)*

(1 mark)

(Running total carried forward: ________)

(Running total carried forward: _________)

Further possible facts, based on primary and secondary features

8	**The firm owns just 3 hammers and at least 5 brushes** *Use the function card and appropriate subsets of tools*

(1 mark)

9	**No plumber of the firm has borrowed a spanner**

(1 mark)

10	**If Bill has borrowed a hammer then Bill is not a painter**

(1 mark)

(Total (max 10): __________)

Please attach the present document to your Answerbook